Base Case Assessment of the South African Sardine Resource

C.L. Cunningham* and D.S. Butterworth*

Introduction

The assessment of the South African sardine resource has been updated from the last assessment (Cunningham and Butterworth 2004) to take account of new data and data adaptations as follows:

i) an update in the time series of November spawner biomass and May recruitment estimates from acoustic surveys, such that the new time series reflects uncapped estimates of biomass based on new target strength calculations, with correction for attenuation throughout (see Cunningham *et al*. 2007a),

ii) new data for 2004 to 2006 which were not included in the last assessment conducted in 2004, and

iii) quarterly proportion-at-length data from the commercial catches (previously annual catch-at-age data, derived using ALKs from Michael Kerstan prior to 1999 and an 'average' ALK from 2000 to 2003, were assumed to be observed without error).

In addition, this assessment has been updated from previous assessments to include:

iv) a plus group of age 5 (previously sardine were assumed to spawn at age 5 and then die),

v) catch now assumed to be taken in a pulse mid-way between each quarter November-January, February-April, May-July and August-October (previously assumed to be taken on 1 May).

vi) estimates of length-at-age and corresponding weights at age that are year-dependent, and

vii) estimation of initial numbers / proportions at age in November 1983, thereby removing all data prior to 1983 which are considered less reliable.

The July 2007 International Stock Assessment Workshop recommended that the survey age- and length-composition data be excluded from the likelihood function for this assessment (Anon. 2007). This was due to the high inter-annual variability of these data and, in particular, an inability to detect the strong year-classes in the age-composition data. Thus, unlike the previous assessment, no ageing data has been used in the assessment.

Population Dynamics Model

Two population dynamics models were used for the base case South African sardine resource. The data used in these assessments are listed in Cunningham *et al*. 2007b. The first assessment, detailed in Appendix A, uses the commercial proportion-at-length data together with a cohort-dependent two-straight

* MARAM (Marine Resource Assessment and Management Group), Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch, 7701, South Africa. Email: c.l.cunningham@telkomsa.net, doug.butterworth@uct.ac.za.

line growth curve to estimate commercial selectivity at age, where this varied by quarter for ages 1 and 2. Selectivity at age 0 is estimated biennially. Given the lack of age data in this assessment, the model struggled to estimate realistic selectivity values and thus a constraint was included in the model so that selectivities at ages 2, 3 and 4 would not differ drastically (see page 22). Given that:

i) a large number of parameters were required in the estimation of the growth curve and selectivities,

ii) the commercial length data were not very informative, leading to estimation problems, and

iii) only selectivities at ages 1 to 5+ are required for OMP testing,

it was decided to use the average (over quarters) selectivities-at-age output from this assessment model as fixed inputs to a more stable assessment which excluded the commercial length data altogether. This assessment is detailed in Appendix B; it is intended to use this to provide the operating models for OMP testing to avoid likely estimation problems with the other assessment procedure for MCMC evaluations of posteriors. In this model, bycatch is assumed to comprise 0 year old fish only, and directed catch 1+ year old fish only.

For both assessments an informative prior was used for the multiplicative bias in the November acoustic survey with the prior distributions for the remaining estimated parameters chosen to be relatively uninformative (see Appendices A and B for details).

Initial testing of the model revealed an incompatibility between relatively low adult natural mortality values and the sharp increase and then decrease recently observed in the sardine 1+ biomass. Allowing the model to estimate adult natural mortality resulted in a value of around 0.8. This is appreciably larger than the value of 0.4 used previously, though this is in part a consequence of the inclusion of a plus group on this occasion. At this stage juvenile natural mortality has been left equal to 1.0 as for the previous assessment. Comparisons between alternative adult and juvenile natural mortality combinations will be presented at a later stage.

Bayesian Estimation

An objective function consisting of the negative log likelihood equations (A.11) and (A.12) for the assessment including the commercial length data, and (B.8) for the assessment excluding the commercial length data (for which fixed commercial selectivity parameters are input), added to the negative logs of the 119 prior distributions for the assessment including the commercial length data and 34 prior distributions for the assessment excluding the commercial length data, was minimised using AD Model Builder (Otter Research Ltd. 2000) to fit the model and estimate the parameters at the posterior mode.

Results

Base Case at Posterior Mode

For ease of comparison, the results from both assessments (that including and that excluding commercial length data) are presented together. The model fit to the data at the posterior mode is shown in Figure 1 for the November acoustic 1+ biomass and Figure 2 for recruitment. Figure 3 shows the residuals from the fit to the quarterly commercial proportion-at-length data. The model predicted November spawner biomass and recruitment at the posterior mode are plotted in Figure 4, together with the model estimated hockey-stick stock-recruitment curve and constant recruitment from 2000 to 2004. The inflection point (b^S) and maximum recruitment (a^S) of the estimated Hockey-Stick curve are lower than the corresponding values estimated by the last assessment (Table 2), though note that comparisons are affected by the changed value for adult natural mortality and refinement of survey estimates of abundance.

The selectivity curves estimated in the assessment including commercial length data, and fixed in the assessment excluding commercial length data, are shown in Figures 5a and b, with values listed in Table 1. The mean lengths at age estimated in the assessment model including commercial length data using the cohort dependent 2-line growth curve under equation (A.8) are shown in Figure 6, with parameters of the fit listed in Table 1.

Implications for the OMP

Key model parameter values and outputs at the posterior mode are given in Table 1. The majority of parameters common to both assessments have similar values, with some differences in the proportion of numbers-at-age in November 1983, that are of minor importance. Thus the inputs to the OMP (albethey posterior distributions of key model parameters and outputs rather than values at the posterior mode) do not change by fixing selectivities-at-ages 1 to 5+ in the manner suggested.

For comparative purposes, Table 2 lists some key model parameters and outputs at the joint posterior mode for the model with selectivity fixed on input, together with those from the last assessment used to develop OMP-04. The average 1+ biomass between 1991 and 1994, used to define risk to develop OMP-04 is 34% lower than previously. Figure 7 shows the November 1+ biomass over time in relation to carrying capacity and the average 1991 to 1994 biomass. The sardine November 1+ biomass has historically dropped below the average 1991 to 1994 biomass 39% of the time (Figure 7). In relation to measurement of risk to the resource, it should be noted that the standard deviation in recruitment residuals is estimated to be appreciably higher than that used to develop OMP-04, while carrying capacity is lower. However, when the assessment model was re-run estimating a different σ_r^S for the peak years of 2000-

2004, the standard deviation in recruitment residuals in non-peak years becomes similar to that used to develop OMP-04, though that for the peak-years is much greater.

Figure 8 shows the harvest rate over time at the posterior mode, calculated as the proportion of observed catch by mass to model predicted 1+ biomass.

Summary

This document has detailed the updated base case assessment of the South African sardine resource and provided results at the posterior mode. The full posterior distributions for this base case are still to be computed. Robustness tests to this base case also await computation. The posterior distributions resulting from the base case hypotheses and some key robustness tests will be used as input into the testing framework for the combined management procedure for sardine and anchovy currently under development.

References

Anon. 2007. Report of the NRF/SA Pelagic and Rock Lobster Industries International Stock Assessment Workshop, University of Cape Town, 9-13 July, 2007. Unpublished report available from MARAM, Department of Mathematics and Applied Mathematics, University of Cape Town. 38pp.

Brandão, A., Butterworth, D.S., Watkins, B.P., and Staverees, L. 2002. An updated assessment of the toothfish (*Dissostichus eleginoides*) resource in the Prince Edward Island vicinity and extensions taking commercial catch-at-length data into account. Unpublished report, CCAMLR. WG/FSA/02-76. 23pp.

Cunningham, C.L., and Butterworth, D.S. 2004. Base Case Bayesian Assessment of the South African Sardine Resource. MCM Document WG/PEL/APR04/02. 22pp.

Cunningham, C.L., Butterworth, D.S. and Coetzee, J. 2007a. The Estimation of South African Sardine and Anchovy Survey Uncapped Biomass From Capped Biomass. *In Prep*.

Cunningham, C.L., van der Westhuizen, J.J., Durholtz D. and Coetzee, J. 2007b. A Record of the Generation of Data Used in the Sardine and Anchovy Assessments. MCM Document MCM/2007/SEPT/SWG-PEL/03. 28pp.

De Oliveira, J.A.A. 2003. The Development and Implementation of a Joint Management Procedure for the South African Pilchard and Anchovy Resources. PhD Thesis, University of Cape Town, South Africa.

Otter Research Ltd. 2000. An Introduction to AD Model Builder Version 4: For Use in Nonlinear Modeling and Statistics. Otter Research Ltd. (http://www.otter-rsch.com/)

Pope, J.G. 1984. The performance of short cut methods for catch forcasts. ICES Document CM 1984/D:3. 12pp.

*Table 1. Key model parameter values and model outputs estimated at the joint posterior mode. Fixed values are given in **bold**. Numbers are reported in billions and biomass in thousands of tonnes.*

Parameter	Including Commercial Length Data	Excluding Commercial Length Data
M_j^S	**1.0**	**1.0**
M_{ad}^S	**0.8**	**0.8**
k_N^S	0.74	0.74
k_r^S	0.35	0.35
k_r^S / k_N^S	0.48	0.48
$\left(\lambda_N^S\right)^2$	0	0
$\left(\lambda_r^S\right)^2$	0	0
$S_{q,1}$, $q=1,...,4$	0.01, 0.09, 1.10, 0.00 (avg = 0.30)	**0.30**
$S_{q,2}$, $q=1,...,4$	1.66, 0.22, 0.85, 0.92 (avg = 0.91)	**0.88**
S_3	0.85	**0.88**
S_4	**1.00**	**1.00**
S_{5+}	0.30	**0.30**
$\overline{L}_{Mean0}$ [1]	5.67	N/A
L_{Diff}	12.78	N/A
m_{2+}	0.13	N/A
ϑ_0^2	5.09	N/A
ϑ_1^2	7.19	N/A
ϑ_2^2	1.45	N/A
ϑ_3^2	1.36	N/A
ϑ_4^2	1.93	N/A
ϑ_{5+}^2	0.00	N/A

[1] $\overline{L}_{Mean0} = \frac{1}{23}\sum_{y=1984}^{2006} L_{Mean0,y}$

Table 1 (continued).

Parameter	Including Commercial Length Data	Excluding Commercial Length Data
N_{1983}^{S}	4.8	4.9
$Nprop_{0}^{S}$	0.56	0.39
$Nprop_{1}^{S}$	0.03	0.09
$Nprop_{2}^{S}$	0.00	0.52
$Nprop_{3}^{S}$	0.21	0.00
$Nprop_{4+}^{S}$	0.21	0.00
$N_{2006,1}^{S}$	14.9	14.9
$N_{2006,2}^{S}$	2.3	2.4
$N_{2006,3}^{S}$	1.2	1.2
$N_{2006,4}^{S}$	3.1	3.0
$N_{2006,5+}^{S}$	3.2	3.2
$\overline{B}_{Nov}^{S}$ 2	595.2	594.7
K_{normal}^{S}	3461.1	3446.3
K_{peak}^{S}	2751.4	2759.1
a^{S}	76.9	76.8
b^{S}	818.8	828.0
c^{S}	75.0	75.6
σ_{r}^{S}	0.755	0.751
η_{2005}^{S}	-0.796	-0.795
s_{cor}^{S}	0.194	0.193

[2] OMP-04 was developed using Risk defined as "the probability that 1+ sardine biomass falls below the average 1+ sardine biomass between November 1991 and November 1994 at least once during the projection period of 20 years".

Table 2. Key parameter values and outputs at the joint posterior mode for the sardine assessment for use in developing the OMP, with comparative values used in developing OMP-04 also shown. Fixed values are given in ***bold****. Biomasses are given in thousands of tonnes and numbers in billions. The final column shows comparative values when an alternative variance in recruitment residuals from 2000-2004 is estimated.*

	Previous Assessment (used to develop OMP-04)		Updated Assessment (fixed selectivity)		
				σ_r^S same for all years	σ_r^S differs for 2000-2004
Starting numbers at age	$N^S_{2003,1}$	29.543	$N^S_{2006,1}$	14.929	18.069
	$N^S_{2003,2}$	22.462	$N^S_{2006,2}$	2.354	1.997
	$N^S_{2003,3}$	15.701	$N^S_{2006,3}$	1.172	1.032
	$N^S_{2003,4}$	7.468	$N^S_{2006,4}$	3.035	3.117
			$N^S_{2006,5+}$	3.168	3.275
Starting spawner biomass	$B^S_{2003,Nov}$	3176	$B^S_{2006,Nov}$	713	713
Directed fishery selectivity	S_1^S [3]	0.662	S_1	**0.30**	**0.30**
	S_2^S	1.000	S_2	**0.88**	**0.88**
	S_3^S	0.857	S_3	**0.88**	**0.88**
	S_4^S	0.331	S_4	**1.00**	**1.00**
			S_{5+}	**0.30**	**0.30**
Juvenile natural mortality	M^S_{ja}	**1.0**	M^S_0	**1.0**	**1.0**
Adult natural mortality	M^S_{ad}	**0.4**	$M^S_a \; a=1,\ldots,5+$	**0.8**	**0.8**
Biases for November and recruitment surveys	k^S_N	0.720	k^S_N	0.738	0.744
	k^S_r	1.039	k^S_r	0.353	0.343
Stock-recruitment parameters	a^S	86.3	a^S	76.8	78.0
	b^S	2319	b^S	828.0	801.8
			c^S	75.6	72.7
	K^S	5926	K^S_{normal}	3446	2871
			K^S_{peak}	2759	4478
Last estimated recruitment residual	η^S_{2002}	0.009	η^S_{2005}	-0.795	-1.08
Recruitment residual standard deviation	σ^S_r	0.416	σ^S_r	0.751	0.409 (non-peak); 1.267 (peak)
Recruitment serial correlation	s^S_{cor}	0.257	s^S_{cor}	0.193	0.313
Average 91-94 Biomass	$\overline{B}^S_{Nov}$	907	$\overline{B}^S_{Nov}$	595	552

[3] Selectivity was estimated as $S_a^S = \overline{F}_a^S / \overline{F}_{\max}^S$, using $\overline{F}_a^S = \frac{1}{5}\sum_{y=1999}^{2003} \frac{C^S_{y,a}}{N^S_{y,a} e^{-M_a^S/2}}$, where $\overline{F}^S_{\max}$ = $\max\{\overline{F}_1^S; \overline{F}_2^S; \overline{F}_3^S; \overline{F}_4^S\}$.

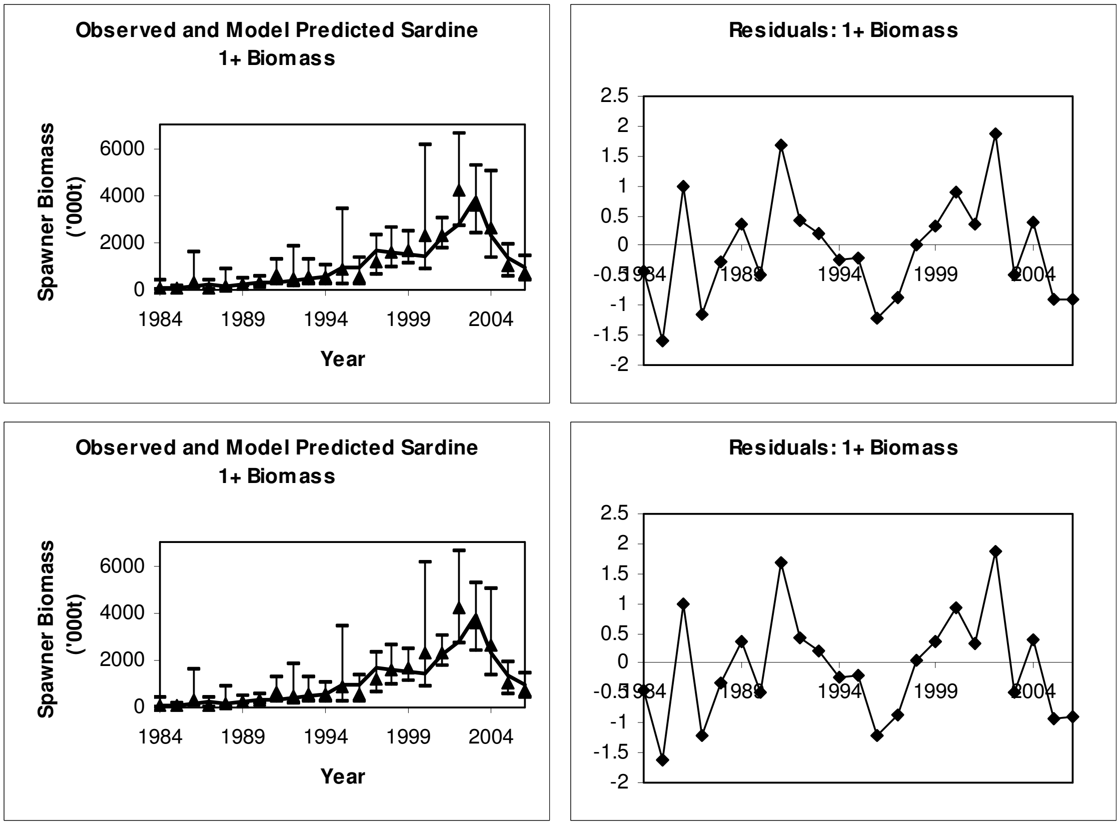

Figure 1. Acoustic survey observed and model predicted November sardine 1+ biomass from 1984 to 2006 for the assessment including commercial length data (upper figures) and the assessment excluding commercial length data (lower figures). The observed indices are shown with 95% confidence intervals. The residuals from the fits are given in the right hand plots.

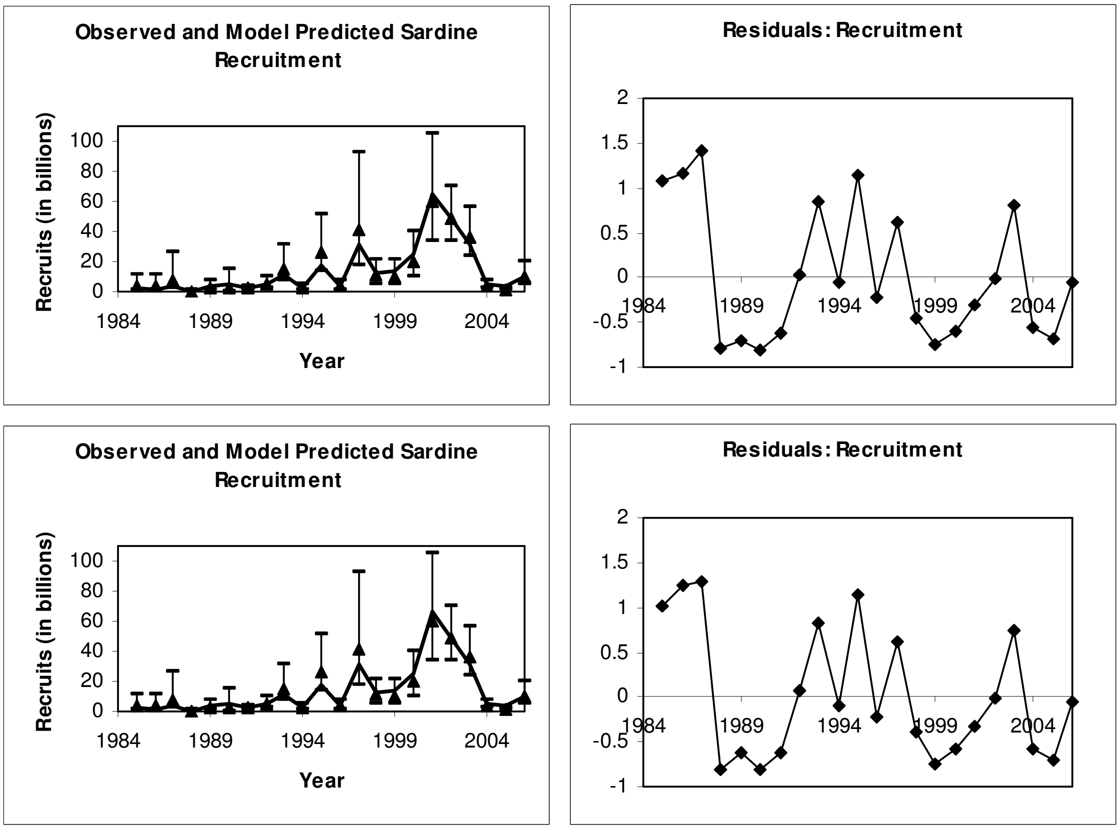

Figure 2. Observed and model predicted sardine recruitment numbers from May 1985 to May 2006 for the assessment including commercial length data (upper figures) and the assessment excluding commercial length data (lower figures). The observed indices are shown with 95% confidence intervals. The residuals from the fits are given in the right hand plots.

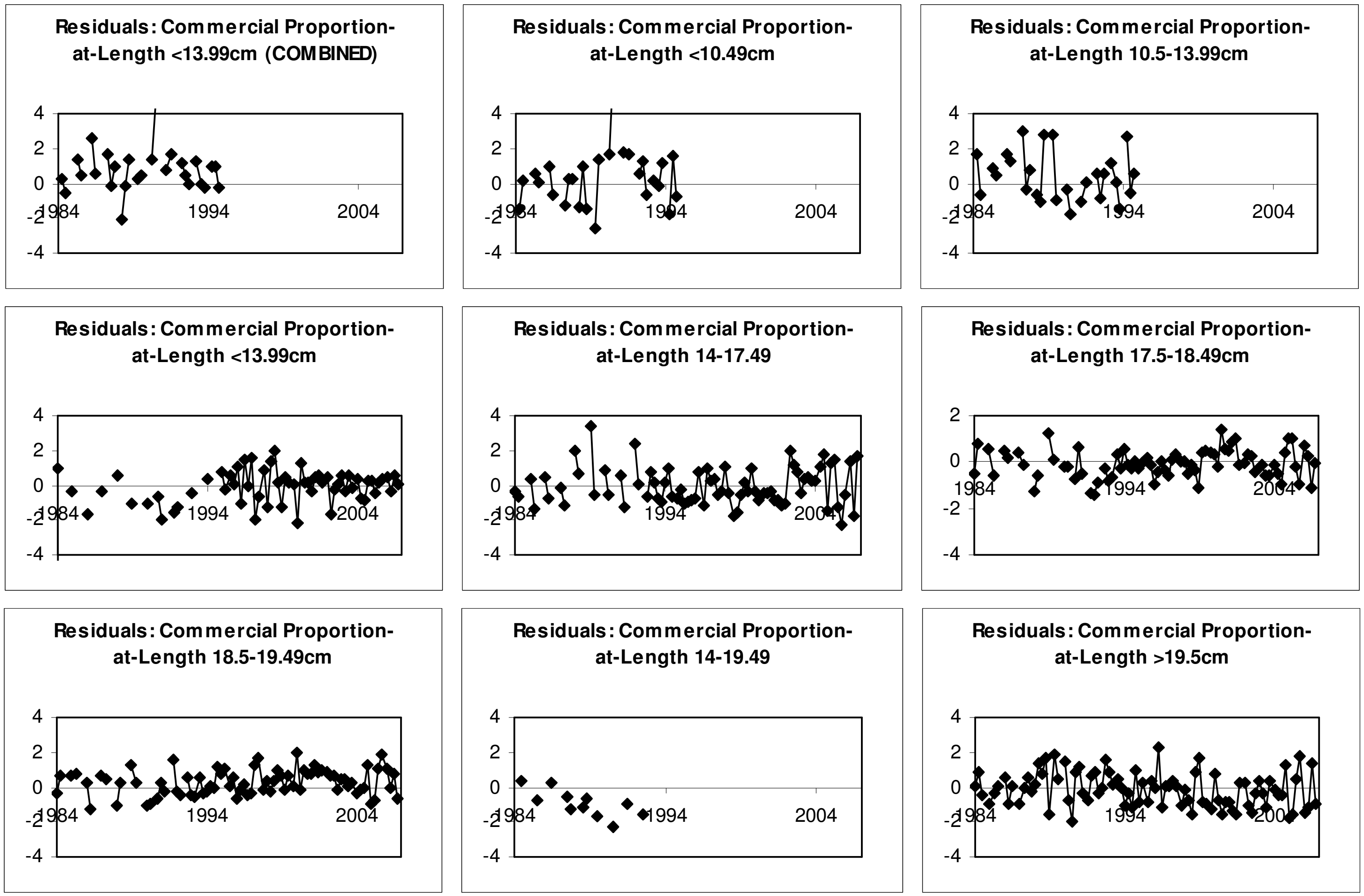

Figure 3. Residuals from the fit of the model predicted proportion-at-length in the commercial catch to the observed data in the assessment which included these data. In some quarters, two or more length classes were combined so that the observed proportion was ≥0.002; thus in addition to the six length classes, residuals are shown for these combined classes of < 13.99cm and 14-19.49cm. The combined residuals for the <10.5cm and 10.5cm-13.99cm length classes are also shown for comparison with the residuals for which only a single data point for both length classes is available.

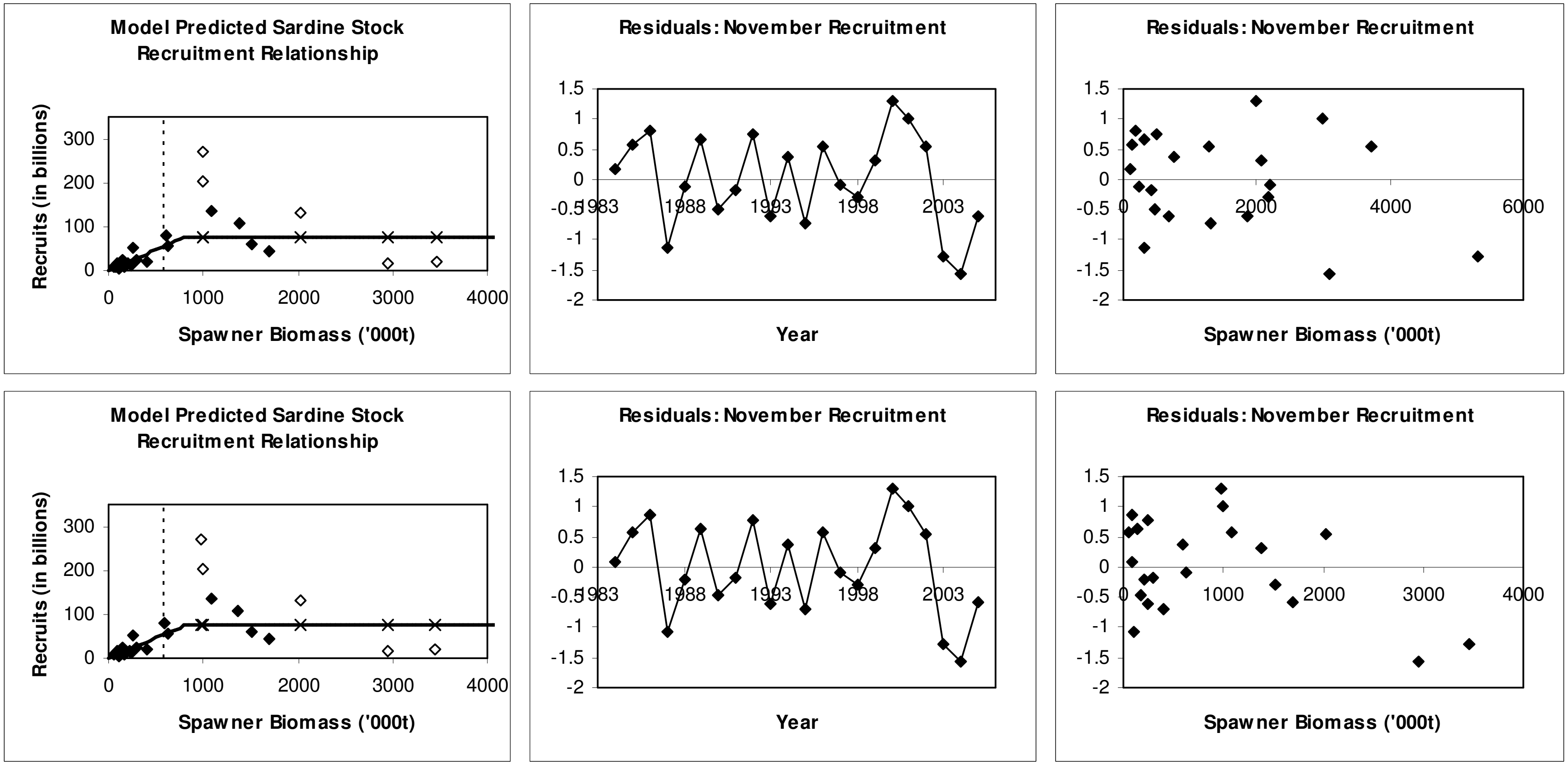

Figure 4. Model predicted sardine recruitment (in November) plotted against spawner biomass from November 1984 to November 2005, with the 'hockey-stick' stock-recruit curve and the constant recruitment between 2000 and 2004 also shown. The open circles denote the 2000 to 2004 November spawner biomass and recruitment. The upper figures are from the assessment including commercial length data and the lower figures are from the assessment excluding commercial length data. The dashed line indicates the average 1991 to 1994 1+ biomass (used in the definition of risk in OMP-04). The residuals from the fit are given in the right hand plots, against year and against spawner biomass.

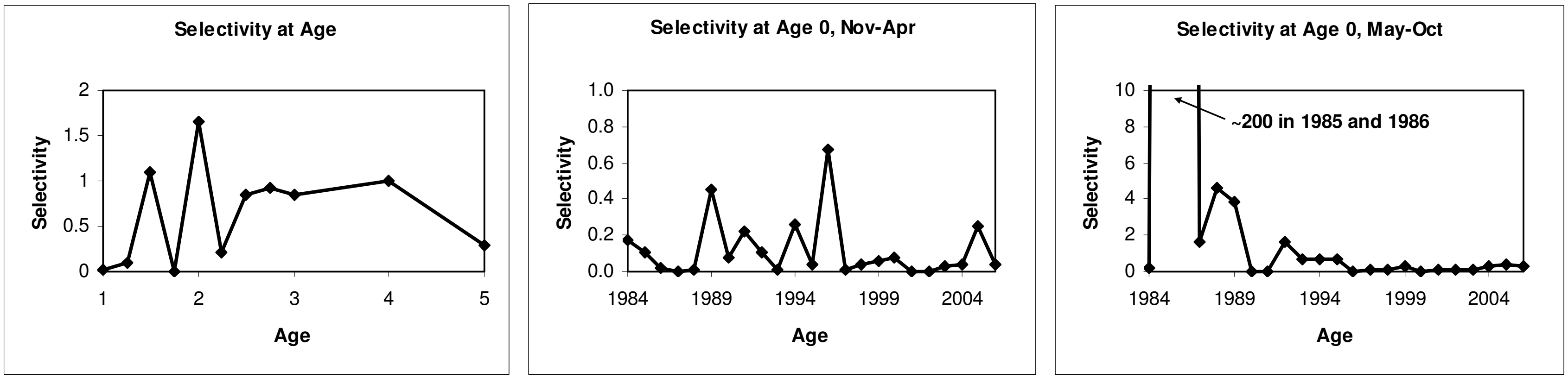

Figure 5a. Quarterly selectivity at ages 1 and 2, annual selectivity at ages 3 to 5+ and biannual selectivity at age 0 estimated by the assessment including commercial length data.

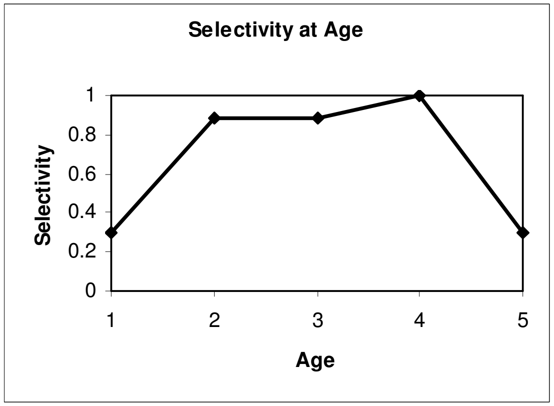

Figure 5b. Annual selectivity at ages 1 to 5+ input into the assessment excluding commercial length data.

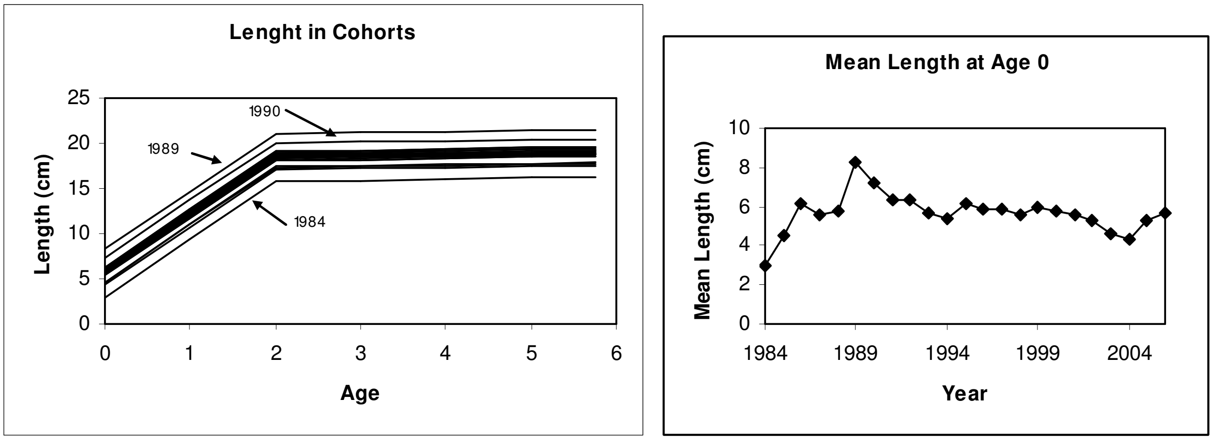

Figure 6. Model predicted length-at-age, which is cohort-dependent. The mean length at age 0 is plotted against year in the right hand figure.

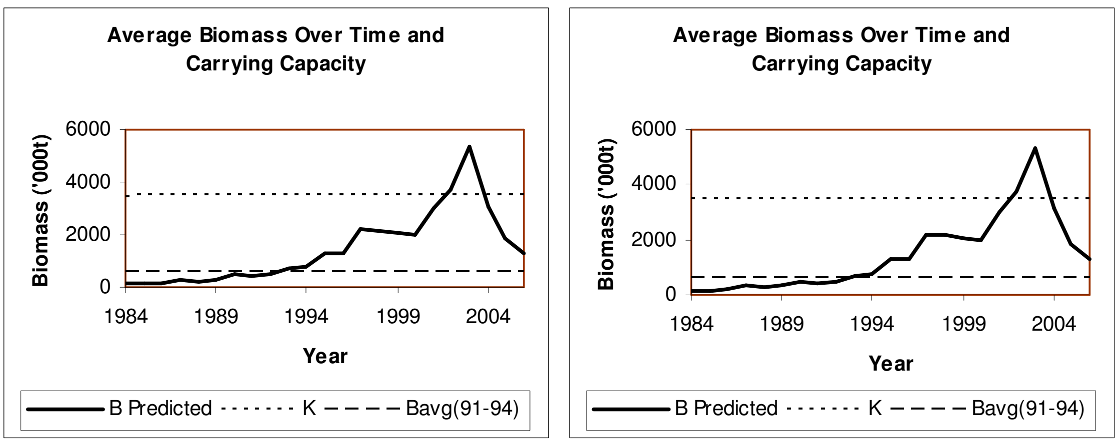

Figure 7. The model predicted November sardine 1+ biomass, plotted against carrying capacity and the average November 1991 to 1994 1+ biomass. This last quantity was used as the risk threshold in developing OMP-04. The left figure results from the assessment including commercial length data and the right figure results from the assessment excluding commercial length data.

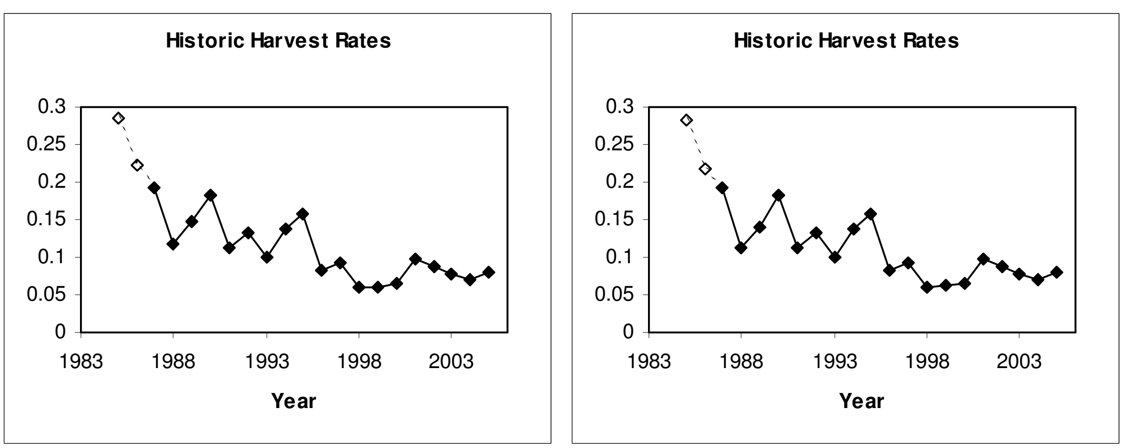

Figure 8. The historic harvest rate (catch by mass to 1+ biomass) on sardine for the assessment including (left) and excluding (right) commercial length data. The first two values are indicated by open symbols as they are less reliably estimated.

Appendix A: Bayesian Assessment Model for the South African Sardine Resource, Including Catch at Length Data

Base Case Model Assumptions

1) All fish have a theoretical birthdate of 1 November.
2) Sardine spawn for the first time (and are called adult sardine) when they turn two years old.
3) A plus group of age five is chosen.
4) Two surveys are held each year: the first takes place in November (known as the November survey) and surveys the adult stock; the second is in May/June (known as the recruit survey) and surveys juvenile sardine (also called recruits or 0-year-old sardine).
5) The November survey provides a relative index of abundance of known bias.
6) The recruit survey provides a relative index of abundance of unknown bias.
7) The survey strategy is such that it results in surveys of invariant bias over time.
8) Pulse fishing occurs four times a year, in the middle of each quarter after the birthdate.
9) Natural mortality is year-invariant for juvenile and adult fish, and age-invariant for adult fish.

Population Dynamics

Numbers-at-age at 1 November

$$N^S_{y,a} = \left(\left(\left(\left((N^S_{y-1,a-1}e^{-M^S_{a-1}/8} - \hat{C}^S_{y,1,a-1})e^{-M^S_{a-1}/4}\right) - \hat{C}^S_{y,2,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,3,a-1}\right)e^{-M^S_{a-1}/4} - \hat{C}^S_{y,4,a-1}\right)e^{-M^S_{a-1}/8}$$

$$y = 1984,\ldots,2006,\ a = 1,\ldots,4$$

$$N^S_{y,5+} = \left(\left(\left(\left((N^S_{y-1,4}e^{-M^S_{4}/8} - \hat{C}^S_{y,1,4})e^{-M^S_{4}/4}\right) - \hat{C}^S_{y,2,4}\right)e^{-M^S_{4}/4} - \hat{C}^S_{y,3,4}\right)e^{-M^S_{4}/4} - \hat{C}^S_{y,4,4}\right)e^{-M^S_{4}/8}$$
$$+\left(\left(\left(\left((N^S_{y-1,5+}e^{-M^S_{5+}/8} - \hat{C}^S_{y,1,5+})e^{-M^S_{5+}/4}\right) - \hat{C}^S_{y,2,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,3,5+}\right)e^{-M^S_{5+}/4} - \hat{C}^S_{y,4,5+}\right)e^{-M^S_{5+}/8}$$

$$y = 1984,\ldots,2006 \qquad \text{(A.1)}$$

where

$N^S_{y,a}$ is the number (in billions) of sardine of age a at the beginning of November in year y;

$\hat{C}^S_{y,q,a}$ is the estimated number (in billions) of sardine of age a caught during quarter q of year y ($q=1$ for November $y-1$ to January y, $q=2$ for February to April y, $q=3$ for May to July y and $q=4$ for August to October y);

M^S_a is the rate of natural mortality (in year^{-1}) of sardine of age a.

Biomass associated with the November survey

$$\hat{B}^S_{y,N} = k^S_N \sum_{a=1}^{5+} N^S_{y,a} w^S_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.2)}$$

where

$\hat{B}^S_{y,N}$ is the biomass (in thousand tonnes) of adult sardine at the beginning of November in year *y*, associated with the November survey;

k^S_N is the constant of proportionality (multiplicative bias) associated with the November survey; and

$w^S_{y,a}$ is the mean mass (in grams) of sardine of age *a* sampled during the November survey of year *y*.

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB^S_{y,N} = \sum_{a=2}^{5+} N^S_{y,a} w^S_{y,a} \qquad y = 1984,\ldots,2006 \qquad \text{(A.3)}$$

Catch

The catch at age by number is calculated using Pope's approximation (Pope 1984):

$$\hat{C}^S_{y,1,a} = N^S_{y-1,a} e^{-M^S_a/8} S_{y,1,a} F_{y,1}\,,\ a = 0,\ldots,5+$$

$$\hat{C}^S_{y,2,a} = \left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} S_{y,2,a} F_{y,2}\,,\ a = 0,\ldots,5+$$

$$\hat{C}^S_{y,3,a} = \left(\left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,2,a} \right) e^{-M^S_a/4} S_{y,3,a} F_{y,3}\,,\ a = 0,\ldots,5+$$

$$\hat{C}^S_{y,4,a} = \left(\left(\left(N^S_{y-1,a} e^{-M^S_a/8} - \hat{C}^S_{y,1,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,2,a} \right) e^{-M^S_a/4} - \hat{C}^S_{y,3,a} \right) e^{-M^S_a/4} S_{y,4,a} F_{y,4}\,,\ a = 0,\ldots,5+ \qquad \text{(A.4)}$$

where

$S_{y,q,a}$ is the commercial selectivity at age a during quarter q of year y, which is assumed to be year-independent for ages 1+ (age 0 landings are mostly bycatch which vary year-to-year independent of the fishing mortality on the older fish in the directed fishery); and

$F_{y,q}$ is the fished proportion in quarter q of year y for a fully selected age class a.

In the above equations the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion is estimated by:

$$F_{y,1} = \frac{C^{ObsTon}_{y,1}}{\sum_{a=0}^{5+}\left(\frac{7}{8}w^S_{y-1,a} + \frac{1}{8}w^S_{y,a+1}\right)N^S_{y-1,a}e^{-M^S_a/8}S_{y,1,a}}$$ [4]

$$F_{y,2} = \frac{C^{ObsTon}_{y,2}}{\sum_{a=0}^{5+}\left(\frac{5}{8}w^S_{y-1,a} + \frac{3}{8}w^S_{y,a+1}\right)\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4}S_{y,2,a}}$$

$$F_{y,3} = \frac{C^{ObsTon}_{y,3}}{\sum_{a=0}^{5+}\left(\frac{3}{8}w^S_{y-1,a} + \frac{5}{8}w^S_{y,a+1}\right)\left(\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,2,a}\right)e^{-M^S_a/4}S_{y,3,a}}$$

$$F_{y,4} = \frac{C^{ObsTon}_{y,4}}{\sum_{a=0}^{5+}\left(\frac{1}{8}w^S_{y-1,a} + \frac{7}{8}w^S_{y,a+1}\right)\left(\left(\left(N^S_{y-1,a}e^{-M^S_a/8} - \hat{C}^S_{y,1,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,2,a}\right)e^{-M^S_a/4} - \hat{C}^S_{y,3,a}\right)e^{-M^S_a/4}S_{y,4,a}} \qquad \text{(A.5)}$$

where

$C^{ObsTon}_{y,q}$ is the observed catch tonnage for quarter q of year y from the RLFs.

Given the predicted proportion-at-age in the quarterly commercial catch

$$\hat{p}^{com}_{y,q,a} = \frac{\hat{C}^S_{y,q,a}}{\sum_{a=0}^{5+}\hat{C}^S_{y,q,a}} \qquad \text{(A.6)}$$

the predicted proportion-at-length is then estimated using a two-line growth equation[5]:

$$\hat{p}^{com}_{y,q,l} = \sum_{a=0}^{5+}\hat{p}^{com}_{y,q,a}A^{com}_{y,q,a,l} \qquad y = 1984,\ldots,2006 \quad \text{(A.7)}$$

where

$A^{com}_{y,q,a,l}$ is the proportion of sardine catch-at-age *a* that fall in the length group l (thus $\sum_{l=1}^{6}A_{y,q,a,l} = 1$ for all ages, quarters and years) in quarter q of year y (the quarterly catch-at-length distributions are split into at most 6 length groups).

The matrix A^{com} is calculated under the assumption that length-at-age is normally distributed about a mean given by a two-line growth equation, (Brandão *et al*. 2002):

$$L^{com}_{y,q,a} \sim N\left(L^{Mean}_{y,q,a}, \vartheta^2_a\right) \qquad \text{(A.8)}$$

---

[4] As no survey weight-at-age is available for 1983, it is assumed that $w^S_{1983,a} = w^S_{1984,a}$, and further that $w^S_{y,0} = 0$.

[5] Initial testing of the model used a von Bertalanffy equation, but this did not allow for satisfactory fits to the observed proportions-at-length, particularly for the older age classes.

where $L_{y,q,a}^{Mean} = \begin{cases} \frac{L_{Diff}}{2}\left(a+0.125+(q-1)/4\right)+L_{Mean0,y-a} & \text{if } a<2 \\ m_{2+}\left(a+0.125+(q-1)/4-2\right)+L_{Mean0,y-a}+L_{Diff} & \text{if } a\geq 2 \end{cases}$

The inflection point at age 2 was chosen after initial testing of the model with independent annual growth curves revealed relatively fast growth prior to age 2 and little growth after age 2. As selectivity is used to calculate quarterly catch which is assumed to be taken in the middle of each quarter, 0.125 is added to age a. Here

$L_{Mean0,y}$ denotes the mean length at age 0 in year y;

L_{Diff} denotes the difference between the mean length at age 2 (inflection point) and age 0;

m_{2+} denotes the slope of the growth curve for ages 2+; and

ϑ_a^2 denotes the variance about the mean length for age a.

Recruitment

For the base case assessment a Hockey Stick stock-recruitment curve is assumed for all years outside of the "peak" years, during which a constant recruitment (i.e. in respect of distribution median and independent of spawning biomass) is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. Thus recruitment in November is given by:

$$N_{y,0}^S = \begin{cases} a^S e^{\varepsilon_y^S} & \text{,if SSB}_{y,N}^S \geq b^S \\ \frac{a^S}{b^S} SSB_{y,N}^S e^{\varepsilon_y^S} & \text{,if SSB}_{y,N}^S < b^S \end{cases} \qquad y = 1984,\ldots,1999,2005,2006$$

$$N_{y,0}^S = c^S e^{\varepsilon_y^S} \qquad y = 2000,\ldots,2004 \quad \text{(A.9)}$$

where

a^S is the maximum recruitment (in billions) (i.e. median of the distribution in question);

b^S is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model;

c^S is the constant recruitment (distribution median) during the "peak" years of 2000 to 2004; and

ε_y^S is the annual lognormal deviation of sardine recruitment.

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$\hat{N}^S_{y,r} = k^S_r \left(\left((N^S_{y-1,0} e^{-M^S_0/8} - \hat{C}^S_{y,1,0}) e^{-M^S_0/4} - \hat{C}^S_{y,2,0} \right) e^{-0.5 t^S_y \times M^S_0/12} - \tilde{C}^S_{y,0bs} \right) e^{-0.5 t^S_y \times M^S_0/12} \quad y = 1984, \ldots, 2006 \quad \text{(A.10)}$$

where

$\hat{N}^S_{y,r}$ is the number (in billions) of juvenile sardine at the time of the recruit survey in year *y*;

k^S_r is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}^S_{y,0bs}$ is the observed number (in billions) of juvenile sardine caught between 1 May and the day before the start of the recruit survey, assuming a 15.5cm cut-off length; and

t^S_y is the time lapsed (in months) between 1 May and the start of the recruit survey in year *y*.

<u>Fitting the Model to Observed Data (Likelihood)</u>

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions. Thus the contribution of the survey abundance data to the negative log-likelihood function is given by:

$$\begin{aligned} -\ln L^{surv} = & \tfrac{1}{2} \sum_{y=1984}^{2006} \left\{ \frac{\left(\ln B^S_{y,N} - \ln(\hat{B}^S_{y,N})\right)^2}{(\sigma^S_{y,Nov})^2 + (\lambda^S_N)^2} + \ln\left[2\pi\left((\sigma^S_{y,Nov})^2 + (\lambda^S_N)^2\right)\right] \right\} \\ & + \tfrac{1}{2} \sum_{y=1985}^{2006} \left\{ \frac{\left(\ln N^S_{y,r} - \ln(\hat{N}^S_{y,r})\right)^2}{(\sigma^S_{y,rec})^2 + (\lambda^S_r)^2} + \ln\left[2\pi\left((\sigma^S_{y,rec})^2 + (\lambda^S_r)^2\right)\right] \right\} \end{aligned} \quad \text{(A.11)}$$

where

$B^S_{y,N}$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass from the November survey in year y, with associated CV $\sigma^S_{y,Nov}$;

$N^S_{y,r}$ is the acoustic survey estimate (in billions) of sardine recruitment numbers from the recruit survey in year *y*, with associated CV $\sigma^S_{y,rec}$; and

$(\lambda^S_{N/r})^2$ is the additional variance (over and above the survey sampling CV $\sigma^S_{y,Nov/rec}$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

The commercial proportions at length from the raised length frequencies are assumed to be lognormally distributed; their contribution to the negative log-likelihood function is given by:

$$-\ln L^{prop} = w_{com} \sum_{y=1984}^{2006} \sum_{q=1}^{4} \sum_{l=1}^{l\max(y,q)} \left\{ \frac{p_{y,q,l}^{com} \left(\ln p_{y,q,l}^{com} - \ln \hat{p}_{y,q,l}^{com} \right)^2}{2\left(\sigma_{com}^S\right)^2} + \ln\left(\frac{\sigma_{com}^S}{\sqrt{p_{y,q,l}^{com}}} \right) \right\}^6 \tag{A.12}$$

where

$p_{y,q,l}^{com}$ is the observed proportion (by number) of the commercial catch in length group l of during quarter q ($q=1$ for Nov-Jan, $q=2$ for Feb-Apr, $q=3$ for May-Jul, $q=4$ for Aug-Oct) of year y;

w_{com} is the weighting applied to the commercial proportion at length data;

σ_{com}^S is the standard deviation associated with the proportion-at-length data in the commercial catch, which is estimated in the fitting procedure by:

$$\sigma_{com}^S = \sqrt{\sum_{y=1984}^{2006} \sum_{q=1}^{4} \sum_{l=1}^{l\max(y,q)} p_{y,q,l}^{com} \left(\ln p_{y,q,l}^{com} - \ln \hat{p}_{y,q,l}^{com} \right)^2 \Big/ \sum_{y=1984}^{2006} \sum_{q=1}^{4} \sum_{l=1}^{l\max(y,q)} 1} .$$

No proportion-at-length was fitted for the fourth quarter in 1984, 1985, 1986 and 1989 as the tonnage landed during this quarter was less than 4% of that for the year. The raw data are recorded by 0.5cm length classes from 3.5cm to 23cm. The data were combined to form six length groups: a minus group of 10.49cm, 10.5cm – 13.99cm, 14.0cm – 17.49cm, 17.5cm – 18.49cm, 18.5cm – 19.49cm and a plus group of 19.5cm. In some quarters, the proportion-at-length in these length groups was small (<2%) so that some length groups were further combined (see Cunningham *et al*. 2007b).

Fixed Parameters

Six parameters were fixed externally in this assessment:

$M_{ju}^S = 1$ and $M_{ad}^S = 0.8$

$w_{com} = 0.05$ There were four data points per year for commercial compared to one per year for the survey data. A higher weighting on the commercial proportion-at-length resulted in an unacceptably poor fit to the November survey, which is considered the most reliable source of information.

$L_{Mean0,2005} = 5.32$ and $L_{Mean0,2006} = \frac{1}{21} \sum_{y=1984}^{2004} L_{Mean0,y}$, as there were insufficient data to estimate these parameters precisely. The value for $L_{Mean0,2005}$ was obtained from initial fits to the model for which overall convergence could not be confirmed,

To remove the confounding with fishing proportion F in equation (A.4), analyses set $S_4 = 1$, with $S_{q,a} = S_a$ for a = 3, 4 and 5+ and $q = 1,\ldots,4$ to stabilize the estimation of selectivity.

---

[6] Although strictly there may be bias in the proportions of commercial length-at-age (as for the survey length-at-

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. The prior pdfs for the recruitment residuals are given by:

$$\varepsilon_y^S \sim N\left(0,\left(\sigma_r^S\right)^2\right) \quad , \qquad y = 1984,\ldots,2005$$

A probability density function (pdf) for the overall bias in the November survey was calculated by drawing ten thousand samples from the individual pdfs for each source of error (I. Hampton pers. comm.), see Table A.1 and Figure A.1 below. In the last assessment, target strength was included as a source of error. Given that the new target strength expression has been used in the survey data, target strength was removed as a source of error from this bias, substantially narrowing the pdf (Figure A.1). There may, however, still be systematic errors relating to the target strength that are unaccounted for in this pdf. These are taken into account through sensitivity tests using alternative k_N^S values. A normal distribution, using the mean and standard deviation of the pdf was used as a prior for k_N^S, i.e. $k_N^S \sim N\left(0.722, 0.078^2\right)$.

Table A.1. Individual error factors for hydroacoustic surveys of sardine spawner biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Random[7]
(Beam factor)	0.75	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Random
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

age), no bias is assumed in this assessment. The effect of such a bias is assumed to be small.

[7] Note that for the purposes of this simulation, ‘random’ and ‘variable’ factors are treated in the same manner.

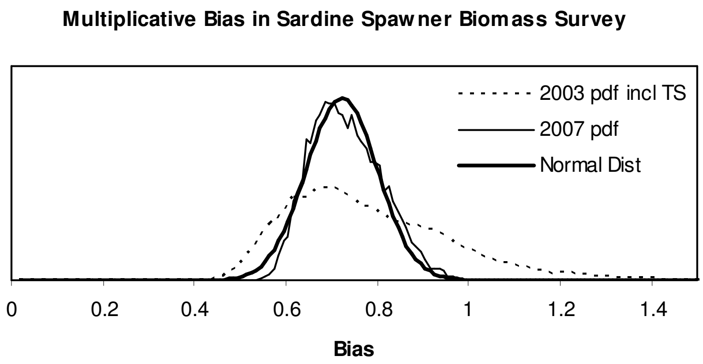

Figure A.1. The probability density function for the overall bias in the sardine November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of error. The normal distribution used as a prior for the bias is also shown. The pdf calculated in 2003 including target strength as a source of error is shown for comparison.

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$\log(k_r^S) \sim U(-100, 0.4)$ (upper bound corresponding to $k_r^S = 1.5$)

$(\lambda_N^S)^2 \sim U(0,10)$

$(\lambda_r^S)^2 \sim U(0,10)$

$\log(a^S) \sim U(0,8)$ (given the lack of *a priori* information on the magnitude of a^S , a log-scale was used)

$\log(c^S) \sim U(0,8)$

$b^S / K^S \sim U(0,1)$

$(\sigma_r^S)^2 \sim U(0.4,10)$

$N_{1983,a}^S = N_{1983} \times Nprop_a$, where $N_{1983} \sim U(0,50)$ billion and $Nprop_a \sim U(0,1)$ for $a = 0, \ldots, 2$ and $Nprop_3 = Nprop_4 = \frac{1}{2}\left(1 - \sum_{a=0}^{2} Nprop_a\right)$.

$S_{y,1,0} \sim U(0,1)$, with $S_{y,2,0} = S_{y,1,0}$, $y = 1984, \ldots, 2006$

$S_{y,3,0} \sim U(0,1)$, with $S_{y,4,0} = S_{y,3,0}$, $y = 1984, \ldots, 2006$

$S_{q,1} \sim U(0,1)$, $q = 1, \ldots, 4$

$S_{q,2} \sim U(0,1)$, $q = 1, \ldots, 4$

$S_3 \sim U(0,1)$, with $S_{q,3} = S_3$ for $q = 1, \ldots, 4$

$S_{5+} \sim U(0,1)$, with $S_{q,5+} = S_{5+}$ for $q = 1, \ldots, 4$

Initial testing of the model revealed almost no difference in selectivity by quarter for ages 3+ so that a single selectivity for each age over all quarters was estimated.

$L_{Mean0,y} \sim U(3,12)$ cm , $y = 1984,\ldots,2004$

$L_{Diff} \sim U(0,15)$ cm

$m_{2+} \sim U(0,1)$ cm.age^{-1}

$\vartheta_a^2 \sim U(0,10)$ cm, for $a = 0,1$

$\vartheta_a^2 \sim U(0,4)$ cm, for $a = 2,\ldots,5+$

One further penalty function was required to stabilize the estimates of selectivity at ages 2 and 3 by smoothing the age dependence towards a quadratic form:

$0.1 * \left(S_{q,2} - 2S_{q,3} + S_{q,4}\right)^2$

<u>Further Outputs</u>

Recruitment serial correlation:

$$s_{cor}^S = \frac{\sum_{y=1984}^{2004} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{2004} \varepsilon_y^2\right)\left(\sum_{y=1984}^{2004} \varepsilon_{y+1}^2\right)}} \tag{A.13}$$

and the standardised recruitment residual value for 2005:

$$\eta_{2005}^S = \frac{\varepsilon_{2005}^S}{\sigma_r^S} \tag{A.14}$$

are also required as input into the OM.

A separate carrying capacity, K^S (essentially the B_N^S value where the replacement line and the stock recruit function intersect) is calculated representing the period of peak abundance (2000 – 2004) to that for the remaining years:

$$K_{normal}^S = a^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{normal,a}^S e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{normal,5+}^S e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1 - e^{-M_{5+}^A}} \right] \tag{A.15}$$

$$K_{peak}^S = c^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{peak,a}^S e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{peak,5+}^S e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1 - e^{-M_{5+}^A}} \right] \tag{A.16}$$

(calculated assuming maximum recruitment in the absence of fishing) where

$\overline{w}_{normal,a}^S$ is the mean mass (in grams) of sardine of age a sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available outside of the peak years (i.e. 1993, 1994, 1996 and 2006).

$\overline{w}^{S}_{peak,a}$ is the mean mass (in grams) of sardine of age a sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available during the peak period (i.e. 2001 - 2004).

The $e^{\frac{1}{2}\left(\sigma_r^S\right)^2}$ factor in the above equation is a bias correction factor, needed given the assumption that recruitment is log-normally distributed about an underlying stock-recruit curve.

Appendix B: Bayesian Assessment Model for the South African Sardine Resource, Excluding Catch At Length Data

Base Case Model Assumptions

These are identical to the assumptions listed in Appendix A.

Population Dynamics

Numbers-at-age at 1 November

$$N_{y,a}^S = \left(\left(\left(\left((N_{y-1,a-1}^S e^{-M_{a-1}^S/8} - \hat{C}_{y,1,a-1}^S)e^{-M_{a-1}^S/4}\right) - \hat{C}_{y,2,a-1}^S\right)e^{-M_{a-1}^S/4} - \hat{C}_{y,3,a-1}^S\right)e^{-M_{a-1}^S/4} - \hat{C}_{y,4,a-1}^S\right)e^{-M_{a-1}^S/8}$$

$$y = 1984,\ldots,2006,\ a = 1,\ldots,4$$

$$N_{y,5+}^S = \left(\left(\left(\left((N_{y-1,4}^S e^{-M_4^S/8} - \hat{C}_{y,1,4}^S)e^{-M_4^S/4}\right) - \hat{C}_{y,2,4}^S\right)e^{-M_4^S/4} - \hat{C}_{y,3,4}^S\right)e^{-M_4^S/4} - \hat{C}_{y,4,4}^S\right)e^{-M_4^S/8}$$
$$+\left(\left(\left(\left((N_{y-1,5+}^S e^{-M_{5+}^S/8} - \hat{C}_{y,1,5+}^S)e^{-M_{5+}^S/4}\right) - \hat{C}_{y,2,5+}^S\right)e^{-M_{5+}^S/4} - \hat{C}_{y,3,5+}^S\right)e^{-M_{5+}^S/4} - \hat{C}_{y,4,5+}^S\right)e^{-M_{5+}^S/8}$$

$$y = 1984,\ldots,2006 \qquad \text{(B.1)}$$

where

$N_{y,a}^S$ is the number (in billions) of sardine of age *a* at the beginning of November in year *y*;

$\hat{C}_{y,q,a}^S$ is the estimated number (in billions) of sardine of age *a* caught during quarter q of year y ($q=1$ for November $y-1$ to January y, $q=2$ for February to April y, $q=3$ for May to July y and $q=4$ for August to October y);

M_a^S is the rate of natural mortality (in year^{-1}) of sardine of age a.

Biomass associated with the November survey

$$\hat{B}_{y,N}^S = k_N^S \sum_{a=1}^{5+} N_{y,a}^S w_{y,a}^S \qquad y = 1984,\ldots,2006 \qquad \text{(B.2)}$$

where

$\hat{B}_{y,N}^S$ is the biomass (in thousand tonnes) of adult sardine at the beginning of November in year *y*, associated with the November survey;

k_N^S is the constant of proportionality (multiplicative bias) associated with the November survey; and

$w_{y,a}^S$ is the mean mass (in grams) of sardine of age *a* sampled during the November survey of year *y*.

Sardine are assumed to mature at age two and thus the spawning stock biomass is:

$$SSB_{y,N}^{S} = \sum_{a=2}^{5+} N_{y,a}^{S} w_{y,a}^{S} \qquad y = 1984,\ldots,2006 \qquad \text{(B.3)}$$

Catch

The catch at age by number is calculated using Pope's approximation for ages 1+ (Pope 1984) and directly from the catch tonnage for age 0:

$$\hat{C}_{y,1,0} = \frac{C_{y,1}^{ObsTon0}}{\frac{7}{8} w_{y-1,0}^{S} + \frac{1}{8} w_{y,1}^{S}}$$

$$\hat{C}_{y,1,a}^{S} = N_{y-1,a}^{S} e^{-M_a^S/8} S_a F_{y,1}\,,\ a = 1,\ldots,5+$$

$$\hat{C}_{y,2,0} = \frac{C_{y,2}^{ObsTon0}}{\frac{5}{8} w_{y-1,0}^{S} + \frac{3}{8} w_{y,1}^{S}}$$

$$\hat{C}_{y,2,a}^{S} = \left(N_{y-1,a}^{S} e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right) e^{-M_a^S/4} S_a F_{y,2}\,,\ a = 1,\ldots,5+$$

$$\hat{C}_{y,3,0} = \frac{C_{y,3}^{ObsTon0}}{\frac{3}{8} w_{y-1,0}^{S} + \frac{5}{8} w_{y,1}^{S}}$$

$$\hat{C}_{y,3,a}^{S} = \left(\left(N_{y-1,a}^{S} e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right) e^{-M_a^S/4} - \hat{C}_{y,2,a}^{S}\right) e^{-M_a^S/4} S_a F_{y,3}\,,\ a = 1,\ldots,5+$$

$$\hat{C}_{y,4,0} = \frac{C_{y,4}^{ObsTon0}}{\frac{1}{8} w_{y-1,0}^{S} + \frac{7}{8} w_{y,1}^{S}}$$

$$\hat{C}_{y,4,a}^{S} = \left(\left(\left(N_{y-1,a}^{S} e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right) e^{-M_a^S/4} - \hat{C}_{y,2,a}^{S}\right) e^{-M_a^S/4} - \hat{C}_{y,3,a}^{S}\right) e^{-M_a^S/4} S_a F_{y,4}\,,\ a = 1,\ldots,5+ \qquad \text{(B.4)}$$

where

S_a is the commercial selectivity at age a, which is assumed to be year-independent for ages 1+; when $S_a = 1$ the age-class a is said to be fully selected;

$F_{y,q}$ is the fished proportion of ages 1+ in quarter q of year y for an age class for which S_a is set equal to 1; and

$C_{y,q}^{ObsTon}$ is the estimated catch tonnage for age 0 in quarter q of year y; this is calculated using the predicted proportion of the observed catch tonnage that is 0 year olds from the assessment in Appendix A.

In the above equations the difference in the year subscript between the catch-at-age and initial numbers-at-age is because these numbers-at-age pertain to November of the previous year.

The fished proportion is estimated by:

$$F_{y,1} = \frac{C_{y,1}^{ObsTon1}}{\sum_{a=1}^{5+}\left(\frac{7}{8}w_{y-1,a}^{S} + \frac{1}{8}w_{y,a+1}^{S}\right)N_{y-1,a}^{S}e^{-M_a^S/8}S_a}$$ [8]

$$F_{y,2} = \frac{C_{y,2}^{ObsTon1}}{\sum_{a=1}^{5+}\left(\frac{5}{8}w_{y-1,a}^{S} + \frac{3}{8}w_{y,a+1}^{S}\right)\left(N_{y-1,a}^{S}e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4}S_a}$$

$$F_{y,3} = \frac{C_{y,3}^{ObsTon1}}{\sum_{a=1}^{5+}\left(\frac{3}{8}w_{y-1,a}^{S} + \frac{5}{8}w_{y,a+1}^{S}\right)\left(\left(N_{y-1,a}^{S}e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4} - \hat{C}_{y,2,a}^{S}\right)e^{-M_a^S/4}S_a}$$

$$F_{y,4} = \frac{C_{y,4}^{ObsTon1}}{\sum_{a=1}^{5+}\left(\frac{1}{8}w_{y-1,a}^{S} + \frac{7}{8}w_{y,a+1}^{S}\right)\left(\left(\left(N_{y-1,a}^{S}e^{-M_a^S/8} - \hat{C}_{y,1,a}^{S}\right)e^{-M_a^S/4} - \hat{C}_{y,2,a}^{S}\right)e^{-M_a^S/4} - \hat{C}_{y,3,a}^{S}\right)e^{-M_a^S/4}S_a} \qquad \text{(B.5)}$$

where

$C_{y,q}^{ObsTon}$ is the estimated catch tonnage for ages 1+ in quarter q of year y. This is calculated using the predicted proportion of the observed catch tonnage that is 1+ year olds from the assessment in Appendix A.

Recruitment

For the base case assessment a Hockey Stick stock-recruitment curve is assumed for all years outside of the “peak” years, during which a constant recruitment (i.e. in respect of distribution median and independent of spawning biomass) is assumed. Recruitment at the beginning of November is assumed to fluctuate lognormally about the stock-recruitment curve. Thus recruitment in November is given by:

$$N_{y,0}^{S} = \begin{cases} a^S e^{\varepsilon_y^S} & \text{,if } SSB_{y,N}^{S} \geq b^S \\ \frac{a^S}{b^S} SSB_{y,N}^{S} e^{\varepsilon_y^S} & \text{,if } SSB_{y,N}^{S} < b^S \end{cases} \qquad y = 1984,\ldots,1999,2005,2006$$

$$N_{y,0}^{S} = c^S e^{\varepsilon_y^S} \qquad y = 2000,\ldots,2004 \quad \text{(B.6)}$$

where

a^S is the maximum recruitment (in billions) (i.e. median of the distribution in question);

b^S is the spawner biomass above which there should be no recruitment failure risk in the hockey stick model;

c^S is the constant recruitment (distribution median) during the “peak” years of 2000 to 2004; and

[8] As no survey weight-at-age is available for 1983, it is assumed that $w_{1983,a}^{S} = w_{1984,a}^{S}$, and further that $w_{y,0}^{S} = 0$.

ε_y^S is the annual lognormal deviation of sardine recruitment (see section on prior distributions).

Number of recruits at the time of the recruit survey

The number of recruits at the time of the recruit survey is calculated taking into account the recruit catch during quarters 1 and 2 (November to April) and an estimate of the recruit catch between 1 May and the start of the survey:

$$\hat{N}_{y,r}^S = k_r^S \left(\left((N_{y-1,0}^S e^{-M_0^S/8} - \hat{C}_{y,1,0}^S) e^{-M_0^S/4} - \hat{C}_{y,2,0}^S \right) e^{-0.5t_y^S \times M_0^S/12} - \tilde{C}_{y,0bs}^S \right) e^{-0.5t_y^S \times M_0^S/12} \quad y = 1984, \ldots, 2006 \quad \text{(B.7)}$$

where

$\hat{N}_{y,r}^S$ is the number (in billions) of juvenile sardine at the time of the recruit survey in year *y*;

k_r^S is the constant of proportionality (multiplicative bias) associated with the recruit survey;

$\tilde{C}_{y,0bs}^S$ is the observed number (in billions) of juvenile sardine caught between 1 May and the day before the start of the recruit survey, assuming a 15.5cm cut-off length; and

t_y^S is the time lapsed (in months) between 1 May and the start of the recruit survey in year *y*.

<u>Fitting the Model to Observed Data (Likelihood)</u>

The survey observations are assumed to be lognormally distributed. The standard errors of the log-distributions for the survey observations of adult biomass and recruitment numbers are approximated by the CVs of the untransformed distributions. Thus the contribution of the survey abundance data to the negative log-likelihood function is given by:

$$\begin{aligned} -\ln L^{surv} = {} & \tfrac{1}{2} \sum_{y=1984}^{2006} \left\{ \frac{\left(\ln B_{y,N}^S - \ln(\hat{B}_{y,N}^S) \right)^2}{(\sigma_{y,Nov}^S)^2 + (\lambda_N^S)^2} + \ln\left[2\pi \left((\sigma_{y,Nov}^S)^2 + (\lambda_N^S)^2 \right) \right] \right\} \\ & + \tfrac{1}{2} \sum_{y=1985}^{2006} \left\{ \frac{\left(\ln N_{y,r}^S - \ln(\hat{N}_{y,r}^S) \right)^2}{(\sigma_{y,rec}^S)^2 + (\lambda_r^S)^2} + \ln\left[2\pi \left((\sigma_{y,rec}^S)^2 + (\lambda_r^S)^2 \right) \right] \right\} \end{aligned} \quad \text{(B.8)}$$

where

$B_{y,N}^S$ is the acoustic survey estimate (in thousands of tonnes) of adult sardine biomass from the November survey in year y, with associated CV $\sigma_{y,Nov}^S$;

$N_{y,r}^S$ is the acoustic survey estimate (in billions) of sardine recruitment numbers from the recruit survey in year *y*, with associated CV $\sigma_{y,rec}^S$; and

$(\lambda_{N/r}^S)^2$ is the additional variance (over and above the survey sampling CV $\sigma_{y,Nov/rec}^S$ that reflects survey inter-transect variance) associated with the November/recruit surveys;

Fixed Parameters

Six parameters were fixed externally in this assessment:

$M^S_{ju}=1$ and $M^S_{ad}=0.8$

$S_1=0.3$ (average over all quarters from the output from the assessment of Appendix A)

$S_2=S_3=0.88$ (average over ages 2 and 3 over all quarters from the output from the assessment of Appendix A; S_2 =0.91 and S_3 = 0.85 were averaged together in order to maintain a single-mode selectivity function)

$S_4=1$ (to remove the confounding with fishing proportion F)

$S_{5+}=0.3$ (output from the assessment of Appendix A)

Estimable Parameters and Prior Distributions

The recruitments are assumed to fluctuate lognormally about the stock-recruitment curve. The prior pdfs for the recruitment residuals are given by:

$$\varepsilon^S_y \sim N\left(0,\left(\sigma^S_r\right)^2\right) \quad , \qquad y=1984,...,2005$$

A probability density function (pdf) for the overall bias in the November survey was calculated by drawing ten thousand samples from the individual pdfs for each source of error (I. Hampton pers. comm.), see Table A.1 and Figure A.1 below. In the last assessment, target strength was included as a source of error. Given that the new target strength expression has been used in the survey data, target strength was removed as a source of error from this bias, substantially narrowing the pdf (Figure A.1). There may, however, still be systematic errors relating to the target strength that are unaccounted for in

Table A.1. Individual error factors for hydroacoustic surveys of sardine spawner biomass, where the values define trapezium form pdfs. Note that these error factors apply to the observed biomass, i.e. they reflect the inverse of the multiplicative bias (applied to predicted biomass) in this document.

Error	Minimum	Likely (lower)	Likely (midpoint)	Likely (upper)	Maximum	Nature
Calibration						
(On-axis sensitivity)	0.90	0.95	1.00	1.05	1.10	Random[9]
(Beam factor)	0.75	0.90	1.00	1.10	1.25	Constant
Surface Schooling	1.00	1.05	1.075	1.10	1.15	Variable
Target Identification	0.50	0.90	1.00	1.10	1.50	Random
Weather Effects	1.01	1.05	1.15	1.25	2.00	Variable

[9] Note that for the purposes of this simulation, 'random' and 'variable' factors are treated in the same manner.

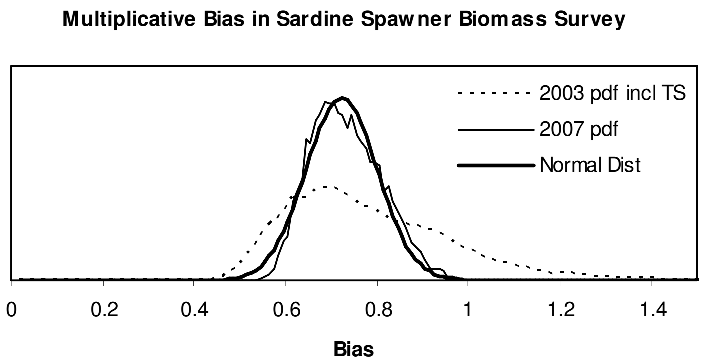

Figure A.1. The probability density function for the overall bias in the sardine November survey, calculated by drawing 10 000 samples from the individual probability distribution functions for each source of error. The normal distribution used as a prior for the bias is also shown. The pdf calculated in 2003 including target strength as a source of error is shown for comparison.

this pdf. These are taken into account through sensitivity tests using alternative k_N^S values. A normal distribution, using the mean and standard deviation of the pdf was used as a prior for k_N^S, i.e. $k_N^S \sim N(0.722, 0.078^2)$.

The remaining estimable parameters are defined as having the following near non-informative prior distributions:

$\log(k_r^S) \sim U(-100, 0.4)$ (upper bound corresponding to $k_r^S = 1.5$)

$(\lambda_N^S)^2 \sim U(0,10)$

$(\lambda_r^S)^2 \sim U(0,10)$

$\log(a^S) \sim U(0,8)$ (given the lack of *a priori* information on the magnitude of a^S, a log-scale was used)

$\log(c^S) \sim U(0,8)$

$b^S / K^S \sim U(0,1)$

$(\sigma_r^S)^2 \sim U(0.4,10)$

$N_{1983,a}^S = N_{1983} \times Nprop_a$, where $N_{1983} \sim U(0,50)$ billion and $Nprop_a \sim U(0,1)$ for $a = 0, \ldots, 2$ and $Nprop_3 = Nprop_4 = \frac{1}{2}\left(1 - \sum_{a=0}^{2} Nprop_a\right)$.

Further Outputs

Recruitment serial correlation:

$$s_{cor}^{S} = \frac{\sum_{y=1984}^{2004} \varepsilon_y \varepsilon_{y+1}}{\sqrt{\left(\sum_{y=1984}^{2004} \varepsilon_y^2\right)\left(\sum_{y=1984}^{2004} \varepsilon_{y+1}^2\right)}} \tag{B.9}$$

and the standardised recruitment residual value for 2005:

$$\eta_{2005}^{S} = \frac{\varepsilon_{2005}^{S}}{\sigma_r^{S}} \tag{B.10}$$

are also required as input into the OM.

A separate carrying capacity, K^S (essentially the B_N^S value where the replacement line and the stock recruit function intersect) is calculated representing the period of peak abundance (2000 – 2004) to that for the remaining years:

$$K_{normal}^{S} = a^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{normal,a}^{S} e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{normal,5+}^{S} e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1-e^{-M_{5+}^{A}}} \right] \tag{B.11}$$

$$K_{peak}^{S} = c^S e^{\frac{1}{2}\left(\sigma_r^S\right)^2} \left[\sum_{a=1}^{4} \overline{w}_{peak,a}^{S} e^{-\sum_{a=0}^{a=1} M_a^S} + \overline{w}_{peak,5+}^{S} e^{-\sum_{a=0}^{4} M_a^S} \frac{1}{1-e^{-M_{5+}^{A}}} \right] \tag{B.12}$$

(calculated assuming maximum recruitment in the absence of fishing) where

$\overline{w}_{normal,a}^{S}$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available outside of the peak years (i.e. 1993, 1994, 1996 and 2006).

$\overline{w}_{peak,a}^{S}$ is the mean mass (in grams) of sardine of age *a* sampled during each November survey, averaged over all November surveys for which an estimate of mean mass-at-age is available during the peak period (i.e. 2001 - 2004).

The $e^{\frac{1}{2}\left(\sigma_r^S\right)^2}$ factor in the above equation is a bias correction factor, needed given the assumption that recruitment is log-normally distributed about an underlying stock-recruit curve.